Optimizing Ballistic Imaging Operations

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Abstract

Ballistic imaging systems can help solve gun crimes by comparing images of cartridge cases, which are recovered from a crime scene or test-fired from a confiscated gun, to a database of images obtained from past crime scenes. However, many U.S. municipalities lack the resources to process (i.e., enter images into the database and search for matches) all of their newly acquired cartridges, and external laboratories are often undercapacitated and generate hits (i.e., matches between new cartridges and database entries) only after lengthy delays. Using data from the Stockton, CA Police Department, which has recently added in-house capacity sufficient to process all cartridges, we analyze two problems: how to allocate capacity when it is limited (i.e., the processing capacity is less than the arrival rate of cartridges) to maximize the number of cartridges that generate at least one hit, and how to prioritize the cartridges that are processed to maximize the usefulness of the hits (i.e., to obtain the hit before the corresponding criminal case is closed). We find that when capacity is limited, the number of cartridges that generate a hit is proportional to only \( \left( \frac{\text{capacity}}{\text{arrival rate}} \right)^2 \), in stark contrast to most manufacturing and service operations, where the output is proportional to \( \frac{\text{capacity}}{\text{arrival rate}} \). However, the number of hits can be significantly increased (e.g., if processing capacity is half of the arrival rate, then the number of hits can be doubled) by prioritizing crime scene evidence over test-fires, and by ranking cartridge types by their hit probability and processing only the higher-ranking cartridge types. We also find that last-come first-served increases the proportion of hits that are useful by only 0.05 relative to first-come first-served, which is probably insufficient to offset the severe inequity introduced by last-come first-served. These results provide a simple and costless way to improve the hit performance of ballistic imaging systems.

**Keywords:** ballistic imaging, operations management, queueing theory, optimization, statistics.
Background

Sixty-nine percent of homicides involve a firearm [1]. For homicides and other gun crimes in which firearms, fingerprints and DNA evidence are not recovered (e.g., the shooter fires from a distance and does not leave the gun at the crime scene), ballistic imaging can be a valuable crime-solving tool. Gun-specific markings are left on the spent cartridge case, hereafter referred to as a cartridge, after a firearm is fired, and ballistic imaging compares the markings on the cartridge to the markings on previously recovered cartridges; ballistic imaging of bullets is also possible but has been much less successful than ballistic imaging of cartridges, and our study restricts itself to cartridges. The National Integrated Ballistic Information Network (NIBIN), developed by The Bureau of Alcohol, Tobacco, Firearms and Explosives (ATF) in 1999 [2], uses computerized imaging technology (in particular, the Integrated Ballistic Identification System (IBIS$^\text{TM}$) created by Forensic Technology, Inc.) to maintain a national database of two- or three-dimensional images of cartridges that are either recovered from crime scenes (referred to here as evidence) or test-fired from confiscated weapons (referred to as test-fires), and computes similarity scores between a newly acquired cartridge and the database entries (although searches are usually restricted to within state or within region). The software system generates a list of (e.g., 10 or 20) possible matches, which are subsequently analyzed by a human examiner to determine whether there are any confirmed hits. Confirmed hits can potentially reveal links between crimes or generate a cold hit between a confiscated weapon and a past crime, both of which can be useful in solving gun crimes. The success of this approach hinges on the established fact that certain firearms tend to be used in multiple gun crimes [3].

While NIBIN has generated more than 47k hits as of early 2012 [4], there has been immense variability across U.S. municipalities in its use and effectiveness: some cities (e.g., Boston [5]) enter all of their cartridges into NIBIN and generate many hits in a timely manner (e.g., before the corresponding criminal case is closed), while others enter very few
cartridges into NIBIN, suffer long delays in processing, and generate very few hits \[2, 4, 6\]. An estimated 72\% of hits in the U.S. come from the 20\% of cities that enter most of their cartridges into NIBIN \[6\], and gun control advocates have argued that ballistic imaging is unreliable, costly and ineffective \[7\]. In an attempt to gain a clearer understanding of ballistic imaging operations, we use several years’ worth of data from Stockton, CA to address three research questions: (i) how does the number of hits vary with the proportion of cartridges that are processed (i.e., entered into NIBIN and compared to database entries in the search for hits), (ii) if there is not sufficient capacity to process all cartridges, which cartridges should be processed to maximize the number of hits, and (iii) given an answer to (ii), in what order should these cartridges be processed to maximize the proportion of hits that are useful (i.e., hits that occur before the corresponding criminal case is closed). The first two research questions are addressed with a mathematical model the determines the number of cartridges that generate at least one hit, and the third research question is addressed with a queueing (i.e., waiting line) model.

Materials and Methods

Because the data guide some of the model development, we begin by describing the data and estimating the parameter values that are needed in the hit model and the queueing model. Then we present the hit model and analyze it under a variety of capacity allocation policies, and finally we describe how the queueing model is used to compute the proportion of hits that are useful.

Data and Parameter Estimation

Description of the Data. Stockton, CA has a population of approximately 300k people, and in 2012 was the second most violent city in CA \[8\]. Their ballistic imaging processes underwent a major change in 2012-2013 when they brought the processes in house, by using firearms technicians (including co-author Mardy Beggs-Cassin) to enter newly acquired cartridges into NIBIN and identify high-confidence candidate hits, and hiring a part-time contract
examiner to make the final determination, thereby generating confirmed hits [8]. Prior to 2013, Stockton – like many municipalities – had relied on a state crime laboratory, which had the capacity to process only a small portion of Stockton’s cartridges and did so with considerable delays [8]. This change in personnel and processes helped generate a three-fold increase in hits and did so with fewer delays [8].

We have two data files from Stockton. The first file contains data on 7710 arriving cartridges that were entered into NIBIN from January 1, 2010 to March 3, 2015. Of these 7710 entries, 1007 had some missing data, and – except for calculating the arrival rate of cartridges – these entries are omitted, leaving 6703 NIBIN entries. Each NIBIN entry is characterized by its crime type and cartridge type. There are \( I = 6 \) crime types (Table 1), consisting of homicides, assaults with a deadly weapon, and other crimes, each either recovered from a crime scene or test-fired from a confiscated firearm. Although there are 63 cartridge types (characterized, e.g., by firearm manufacturer and caliber) in this data file, we retain only the 11 types that represent 1% or more of the total entries, and combine the other 52 cartridge types into a single type called “other,” leaving \( J = 12 \) cartridge types (Table 1). For each NIBIN entry, we also know the event date, which corresponds to the crime date for evidence and the gun confiscation date for test-fires; the entry date, which is the date of the NIBIN entry; and the completion date, which is the date that the process of searching for high-confidence hits is completed.

The second data file contains all high-confidence hits detected between July 26, 2011 - February 20, 2013 and June 18, 2013 - March 3, 2015; i.e., data pertaining to four months of hits are missing. There were 1037 hits in total, but 24 of these were later determined not to be a true hit by the examiner and were omitted from the analysis. Of the remaining 1037 - 24 = 1013 hits, 49 had database matches that matched crimes from other cities for which we have no further information. For the remaining 1013 - 49 = 964 hits, we know the crime type, cartridge type (a hit always involves a match of two cartridges of the same type) and
entry date of the matching database entry. Some cartridges generate multiple matches in
the database, and the 1013 hits come from 528 cartridges that generated at least one hit.

Parameters for the Hit Model. From these two data files, we need to calculate $a_{ij}$ (mnemonic
for arrivals), which is the proportion of all arriving cartridges that are of crime type $i$ and
cartridge type $j$, and $h_{ij}$ (mnemonic for hits), which is the proportion of all arriving cartridges
of crime type $i$ and cartridge type $j$ that generated at least one hit. We address two issues
while estimating $a_{ij}$ and $h_{ij}$: missing hit data and small sample sizes.

Recall that the hit data cover the time periods from July 26, 2011 to February 20,
2013 and from June 18, 2013 to March 3, 2015. To deal with the missing hit data from
February 21, 2013 to June 17, 2013 when estimating $a_{ij}$ and $h_{ij}$, we consider only the
4147 NIBIN entries during July 26, 2011 to February 20, 2013 and from June 18, 2013 to
March 3, 2015, and assume that the only hits generated by these 4147 cartridges are those
generated by the 528 cartridges that had at least one hit in the data file of hits. Hence, the
overall hit proportion, i.e. the proportion of arrivals with at least one hit, is estimated to
be $\frac{528}{4147} = 0.127$, which may slightly underestimate the true hit proportion because of the
time lag between the NIBIN entry date and the completion date. The $a_{ij}$ values (Table 2)
are initially computed as the proportion of the 4147 arrivals that are of crime type $i$ and
cartridge type $j$, and the $h_{ij}$ values (Table 3) are initially computed as the proportion of the
4147 arrivals of crime type $i$ and cartridge type $j$ that generate at least one hit. However,
of the $6 \times 12 = 72$ $(i,j)$ pairs, 43 have less than 30 NIBIN entries (Table 4). So as not to
be overly influenced by these 43 type pairs, we set $a_{ij}$ in Table 2 for these 43 type pairs
all equal to the same number (0.002821), which is the proportion of all NIBIN entries that
are of these 43 type pairs, divided by 43. Similarly, we set $h_{ij}$ in Table 3 for these 43 type
pairs all equal to the same number (0.087475), which is the proportion of all NIBIN entries
of these 43 type pairs that have generated at least one hit. With $a_{ij}$ and $h_{ij}$ in hand, we
compute the average hit proportions for each crime type $i$ $(\sum_{j=1}^{J} a_{ij} h_{ij} / \sum_{j=1}^{J} a_{ij})$ and each
cartridge type \( j \) \( \left( \sum_{i=1}^{I} a_{ij} h_{ij} / \sum_{i=1}^{I} a_{ij} \right) \), which appear in Fig. 1.

Among the entries that generate at least one hit, we assume that the random number of hits generated by an entry, \( X \), follows a geometric distribution, \( P(X = k) = q(1 - q)^{k-1} \) for \( k = 1, 2, \ldots \), where the maximum likelihood estimate for \( q \) is the total number of hits (1013) divided by the total number of cartridges that generated at least one hit (528), which yields \( q = 0.521 \) (Fig. 2). This corresponds to an average of \( \frac{1}{q} = 1.92 \) hits for each arriving cartridge that generates at least one hit.

Our analysis also requires us to estimate the conditional probability that when an arriving cartridge of type \( (i, j) \) has a hit, its database match is of type \( (k, j) \) (recall that matches can only occur between cartridges of the same cartridge type). Furthermore, because crime types 4, 5 and 6 correspond to test-fires from confiscated guns, we expect this conditional probability to equal zero for \( k = 4, 5, 6 \); i.e., if arriving cartridges are processed on a first-come first-served basis, then an arriving cartridge should never match with a test-fire cartridge in the database. We have data for 964 match-hit pairs (i.e., \( (i, j, k) \) triplets), and only 14 of these pairs involve \( k = 4, 5, 6 \), which justifies our decision to set this conditional probability equal to zero for \( k = 4, 5, 6 \). From the remaining 964-14 = 950 hits, we compute \( n_{ijk} \), which is the number of arriving cartridges of type \( (i, j) \) that match a type \( (k, j) \) cartridge in the database, where \( \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{3} n_{ijk} = 950 \) (Table 5). While the natural choice for this conditional probability is \( \frac{n_{ijk}}{\sum_{k=1}^{3} n_{ijk}} \), there are only 950 match-hit pairs to allocate to 216 different \( (i, j, k) \) triplets, and many of the \( n_{ijks} \) in Table 5 equal zero, which makes this approach problematic. Moreover, if we assume that this conditional probability is independent of the crime type \( i \) of the arriving cartridge, and estimate it by \( \frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{i=1}^{I} \sum_{k=1}^{3} n_{ijk}} \), then it still suffers from too many zeroes. Therefore, to smooth these conditional probabilities, we consider two possibilities, \( \frac{a_{kj}}{\sum_{l=1}^{I} a_{lj}} \) and \( \frac{a_{kj} h_{kj}}{\sum_{l=1}^{I} a_{lj} h_{lj}} \), i.e., the proportion of cartridge type \( j \) arrivals that are of crime type \( k \) and the proportion of cartridge type \( j \) hits that are of crime type \( k \), and determine which one is closer to the empirical values, \( \frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{i=1}^{I} \sum_{k=1}^{3} n_{ijk}} \). The
The former estimate is closer to the empirical values using the total variation metric,

\[ \sum_{k=1}^{3} \sum_{j=1}^{J} \left| \frac{a_{kj}}{\sum_{l=1}^{3} a_{lj}} - \frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{l=1}^{3} \sum_{i=1}^{I} n_{ijl}} \right| = 8.83 < \sum_{k=1}^{3} \sum_{j=1}^{J} \left| \frac{a_{kj} h_{kj}}{\sum_{l=1}^{3} a_{lj} h_{lj}} - \frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{l=1}^{3} \sum_{i=1}^{I} n_{ijl}} \right| = 9.50, \]  

(1)

and the Hellinger metric,

\[ \sum_{k=1}^{3} \sum_{j=1}^{J} \left( \sqrt{\frac{a_{kj}}{\sum_{l=1}^{3} a_{lj}}} - \sqrt{\frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{l=1}^{3} \sum_{i=1}^{I} n_{ijl}}} \right)^2 = 3.62 < \sum_{k=1}^{3} \sum_{j=1}^{J} \left( \sqrt{\frac{a_{kj} h_{kj}}{\sum_{l=1}^{3} a_{lj} h_{lj}}} - \sqrt{\frac{\sum_{i=1}^{I} n_{ijk}}{\sum_{l=1}^{3} \sum_{i=1}^{I} n_{ijl}}} \right)^2 = 4.04. \]  

(2)

Therefore we assume that the conditional probability that when a NIBIN entry of type \((i, j)\) has a hit, its database match is of type \((k, j)\) equals

\[ \begin{cases} 
\frac{a_{kj}}{\sum_{l=1}^{3} a_{lj}} & \text{if } k = 1, 2, 3; \\
0 & \text{if } k = 4, 5, 6, 
\end{cases} \]  

(3)

which is given in Table 6.

Parameters for the Queueing Model. Although our analysis of hits requires only the parameters \(a_{ij}, h_{ij}\) and \(q\), and the arrival rate of newly acquired cartridges (to be estimated below), the analysis of useful hits requires us to construct a queueing model and estimate the model’s parameters (Table 7) under two scenarios: the in-house scenario and the external lab scenario. Data for the in-house scenario are taken from Stockton and data for the external lab scenario are taken from [4]. In each scenario, we consider a queueing model with \(m\) servers (i.e., firearms technicians or examiners) working in parallel, where processing entails two tasks: entering newly acquired cartridges into NIBIN and searching for high-confidence hits. The queueing model assumes that the time between consecutive arrivals (an arrival occurs when a crime is committed or a gun is confiscated) is a random variable, and the time to process each arrival is a random variable. The queueing model predicts the steady-state distribution of the total time a customer spends in the system (i.e., in queue plus in service), \(W\), which is the number of days between the event date and the completion date, in terms of four parameters: the number of servers \(m\), the arrival rate \(a\) (i.e., the mean number of
cartridges arriving per day), the processing rate \( p \) (i.e., the reciprocal of the mean amount of time it takes a technician or examiner to enter a new cartridge into NIBIN and search for high-confidence hits), and the variability parameter \( v \), which is the sum of the squared coefficient of variation of the interarrival times \( (c_a^2) \) and the squared coefficient of variation of processing times \( (c_s^2) \), divided by two, where the coefficient of variation of a random variable is its standard deviation divided by its mean. That is,

\[
v = \frac{c_a^2 + c_s^2}{2},
\]

and when we simulate the queueing models, we assume that \( c_a^2 = c_s^2 = v \). We assume that the processing time distribution is the same for all \((i, j)\) type pairs, which implies that the mean time in the system is independent of the precise order in which arrivals are processed, as long as the \( m \) servers are each busy when there is a cartridge that is waiting to be processed.

We begin by estimating the parameters for the in-house scenario. A widely used approximation in queueing theory [9] is that the mean time in the system in steady state is

\[
E[W] = \frac{v}{mp} \left( \frac{\left( \frac{a}{mp} \right) \sqrt{2(m+1)-1}}{1 - \frac{a}{mp}} \right) + \frac{1}{p},
\]

Our approach is to estimate \( a \), \( p \) and \( m \) directly, and use (5) to estimate \( v \) from the empirical value of \( E[W] \) observed in the data. Summing up all entries that had an event date between January 1, 2010 and February 24, 2015 and dividing by 1881 days yields the arrival rate \( a = 4.10/\text{day} \). We also note that the annual arrival rate is fairly steady throughout this five-year period (Fig. 3).

The average rate at which arrivals were entered into NIBIN varied over the five-year period (Fig. 4) because Stockton switched from using a state laboratory to using in-house technicians. However, the NIBIN entries were quite steady after December 25, 2012 (which corresponds to the vertical line in Fig. 4), which roughly coincides with the in-house switch. During the 799 days after December 25, 2012, 4436 arrivals were entered into NIBIN (some
of which had been backlogged for several years), and approximately the same number of entries underwent the search for high-confidence hits. Assuming that the technician was busy throughout this period (which is consistent with working off the backlog), we estimate that the service rate is $p = \frac{4436}{799} = 5.55$ day.

The data on waiting times (i.e., from event date to completion date) of all entries with event dates after December 25, 2012 can be broken into two portions: from event date to NIBIN entry date, and from entry date to completion date. The histogram of the first portion has a very large right tail (Fig. 5) because some of these entries were in a years-old backlog that the technician was initially faced with. That is, the right tail is not caused by current scarce capacity, but by limited capacity prior to December 25, 2012. Consequently, we truncate this distribution at 40 days (i.e., discard all waiting time portions greater than 40 days), leaving a mean delay of 14.14 days between event date and entry date. The histogram for the second portion of the waiting time has several entries with delays of several months (Fig. 6). These isolated long delays are also not due to limited processing capacity, but rather to other issues, and we truncate this distribution at 25 days to get a mean of 3.85 days.

Hence, we assume that $E[W] = 14.14 + 3.85 = 17.99$ days. Stockton had two technician during the period of study; although both technicians often worked on separate days, they sometimes worked simultaneously, and so the true value of $m$ was between one and two (although equation (5) assumes an integer number of servers). Substituting $m = 1$ and our values of $W$, $a$ and $p$ into (5) yields $v = 35.05$. Substituting $m = 2$ into (5) yields $v = 40.17$. Our qualitative results are not affected by our value of $m$, and we hereafter assume that $m = 1$ and $v = c_a^2 = c_s^2 = 35.05$ for the in-house scenario.

For the external lab scenario, we use data from 19 different labs (page 65 in [4]) that specify the mean and standard deviation of the delay at a lab from the date of obtaining evidence (which is typically within several days of the event) to the date of providing confirmed evidence.
hit information. The mean of the 19 mean delays is 337 days and the mean of the 19 coefficients of variation (i.e., the standard deviation divided by the mean) is 1.275. Hence, we assume that the waiting time has mean \( E[W] = 337 \) days and variance \( \text{Var}[W] = (337 \times 1.275)^2 \).

We assume that the lab uses a first-come first-served (FCFS) policy because each municipality that it serves is likely to send the lab its highest-priority evidence, and there is no reason to presume that the lab systematically favors some municipalities over others. We set \( p = 5.55/\text{day}, \) as in the in-house scenario.

To estimate the number of servers, \( m, \) in the external lab scenario, we use data from two sources. The median number of firearms examiners among 109 surveyed NIBIN sites is two (Table 5 in [4]). Because the sites surveyed in [4] include internal and external labs, we also consider a census of publicly funded forensic crime laboratories [10], which states that the mean number of staff is 32 (Table 12 in [10]), approximately 60% of staff are examiners (Table 13 in [10]), and 3% of all completed requests are for firearms (Table 5 in [10]). If the processing times were the same across types of requests in Table 5 in [10], then the mean number of firearms examiners at a publicly funded lab would be \( 32 \times 0.6 \times 0.03 = 0.576. \)

Although we do not know how the mean processing times of firearms compares to those of the other request types, and we do not know whether each of these labs has at least one firearms examiner, these data confirm that the mean number of examiners is not likely to be much larger than two. Taken together, we set \( m = 2, \) leaving us to estimate two parameters, \( a \) and \( v, \) using two equations. The first equation is (5) with \( E[W] = 337 \) days, and the second equation is \( \text{Var}[W] = (337 \times 1.275)^2, \) where by equations (2.3)-(2.4), (3.9)-(3.11) and (4.2)-(4.5) of [9] and letting \( \rho = \frac{a}{mp} \) and \( \Phi(x) \) be the standard normal cumulative distribution function (CDF),

\[
\text{Var}[W] = \left( \frac{v}{mp} \sqrt{\frac{2(m+1)-1}{1-\rho}} \right)^2 \left( \frac{2\rho + \frac{4(1-\rho)v}{v+1}}{\Phi(W_q > 0)} \right) + \frac{v}{p^2}, \tag{6}
\]
where

$$P(W_q > 0) = \min\left\{ \rho^2 \min\left\{ 1, \frac{1 - \Phi\left(\frac{(v+1)\sqrt{m}(1-\rho)}{2v}\right)}{1 - \Phi(\sqrt{m}(1-\rho))} \right\}, \frac{2\rho^2}{1 + \rho} \right\}$$

Solving these two equations jointly yields \( a = 9.39/\text{day} \) and \( v = 735 \).

Finally, to determine whether a hit is useful or not requires us to compare the waiting time to the case closing time \( C \), which is the time interval between the event date and the date that the criminal case is closed or cleared by investigators. For lack of more general data, we use data from homicides, stating that 61.3% of homicides are solved, and of those that are solved, 46.3% are solved in less than one day, 31.5% are solved in 1-7 days, 9.0% in 8-30 days, 9.6% in 1-6 months, and 3.6% in > 6 months (page 159 in [11]). Assuming that unsolved cases are left open (or cold) indefinitely, we let the case closing time \( C \) be infinite with probability 0.387, and we use nonlinear least-squares regression to fit Weibull, lognormal and gamma distributions to these data when \( C < \infty \). The lognormal provides the best fit (Fig. 7), and we assume \( C|C < \infty \) has a lognormal distribution with parameters \( \mu = 0.176 \) and \( \sigma = 2.62 \).

**Hit Analysis**

We define the hit proportion to be the proportion of all arriving cartridges that generate at least one hit. Multiplying the hit proportion by the total arrival rate of cartridges gives the number of cartridges per day that generate at least one hit. Hence, maximizing the hit proportion is equivalent to maximizing the number of cartridges that generate at least one hit over any given time period. The hit proportion depends on \( x_{ij} \), which is the rate that cartridges of crime type \( i \) and cartridge type \( j \) are processed. These capacity allocations need to satisfy \( 0 \leq x_{ij} \leq a_{ij}a \). If there is ample capacity to process all arrivals (i.e., \( x_{ij} = a_{ij}a \) for all \((i, j)\) pairs), then the hit proportion is

$$\sum_{i=1}^{I} \sum_{j=1}^{J} a_{ij} h_{ij}, \quad (8)$$
which we refer to as the potential hit proportion. If capacity is limited (i.e., $x_{ij} < a_{ij}a$ for at least one $(i, j)$ pair), then the hit proportion may be less than in (8) and depends on which of the arrivals are processed. We begin by computing the hit proportion under a generic capacity allocation policy $x_{ij}$.

**Generic Policy:** Because an arriving cartridge may have more than one match in the database, we need to condition on the (geometric) number of matches:

\[
P(\text{type } (i, j) \text{ arrival generates at least one hit})
\]

\[
= P(\text{type } (i, j) \text{ arrival is entered})
\times \sum_{n=1}^{\infty} P(\text{at least one of } n \text{ matches is entered } | \text{ type } (i, j) \text{ arrival has } n \text{ matches})
\times P(\text{type } (i, j) \text{ arrival has } n \text{ matches}).
\]

(9)

Turning to the individual probabilities in (9), we know that

\[
P(\text{type } (i, j) \text{ arrival is entered}) = \frac{x_{ij}}{a_{ij}a},
\]

(10)

and

\[
P(\text{type } (i, j) \text{ arrival has } n \text{ matches}) = (1 - q)^{n-1}qh_{ij}.
\]

(11)

We can express the conditional probability on the right side of (9) as

\[
P(\text{at least one of } n \text{ matches is entered } | \text{ type } (i, j) \text{ arrival has } n \text{ matches})
\]

\[
= 1 - P(\text{no matches entered } | \text{ type } (i, j) \text{ arrival has } n \text{ matches}),
\]

\[
= 1 - \prod_{l=1}^{n} P(\text{an arrival is not entered } | \text{ this arrival is a match of type } (i, j)),
\]

\[
= 1 - [1 - \sum_{k=1}^{l} P(\text{an arrival is entered } | \text{ this arrival is type } (k, j) \text{ and match of type } (i, j))
\times P(\text{this arrival is type } (k, j) | \text{ this arrival is a match of type } (i, j))]^{n},
\]

\[
= 1 - \left[1 - \sum_{k=1}^{3} \frac{x_{kj}a_{kj}}{a_{ij}a^3} \sum_{l=1}^{3} a_{lj}\right]^{n} \text{ by (3)},
\]

\[
= 1 - \left[1 - \frac{3}{a \sum_{l=1}^{3} a_{lj}}\right]^{n}.
\]

(12)
Substituting (10)-(12) into (9) yields

\[ P(\text{type } (i,j) \text{ arrival generates at least one hit}) \]
\[ = \frac{x_{ij}}{a_{ij}} \sum_{n=1}^{\infty} \left(1 - \left[1 - \sum_{k=1}^{3} \frac{x_{kj}}{a \sum_{l=1}^{3} a_{lj}}\right]n\right)(1 - q)^{n-1}qh_{ij}, \]
\[ = \frac{x_{ij}h_{ij}}{a_{ij}a} \left[ 1 - \sum_{n=1}^{\infty} (1 - q)^{n-1}q \left(1 - \sum_{k=1}^{3} \frac{x_{kj}}{a \sum_{l=1}^{3} a_{lj}}\right) n\right], \]
\[ = \frac{x_{ij}h_{ij}}{a_{ij}a} \frac{qa \sum_{k=1}^{3} x_{kj}}{qa \sum_{l=1}^{3} a_{lj} + (1 - q) \sum_{k=1}^{3} x_{kj}}. \] (13)

Equation (13) implies that the hit proportion is

\[ P(\text{an arrival generates at least one hit}) \]
\[ = \sum_{i=1}^{I} \sum_{j=1}^{J} P(\text{an arrival is of type } (i,j)) P(\text{type } (i,j) \text{ generates at least one hit}), \]
\[ = \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij}h_{ij} \frac{\sum_{k=1}^{3} x_{kj}}{qa \sum_{l=1}^{3} a_{lj} + (1 - q) \sum_{k=1}^{3} x_{kj}}. \] (14)

We now use equation (14) to derive the hit proportion for ten different capacity allocation policies, i.e., policies that determine exactly which arrivals of each \((i, j)\) pair to process (Table 8), as the total processing capacity \(p\) varies from 0 to \(a\). Although the hit proportion in (14) depends on \(x_{ij}\), for notational simplicity, for each policy we denote the hit proportion when the total processing capacity is \(p\) by \(h(p)\).

**Random Policy:** The random policy, where each arriving cartridge is entered into NIBIN with probability \(\frac{p}{a}\), is the natural benchmark. Any policy – including, in particular, the first-come first-served (FCFS) policy – that uses neither crime type nor cartridge type information to decide which arriving cartridges are entered into NIBIN will achieve the same hit proportion performance as the random policy. We have \(x_{ij} = a_{ij}p\) for this policy, and substituting this value into (14) yields

\[ h(p) = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{a_{ij}ph_{ij}}{a} \frac{\sum_{k=1}^{3} a_{kj}p}{qa \sum_{l=1}^{3} a_{lj} + (1 - q) \sum_{k=1}^{3} a_{kj}p}, \] (15)
\[ = \frac{(\frac{p}{a})^2}{q + (1 - q)\frac{p}{a}} \sum_{i=1}^{I} \sum_{j=1}^{J} a_{ij}h_{ij}, \] (16)
which reduces to \((\frac{p}{a})^2\) times the potential hit proportion in (8) when \(q = 1\) (i.e., an arriving cartridge has at most one match in the database).

**Optimal Policy**: By (14), the optimization problem is

\[
\max_{x_{ij}} \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij} \frac{h_{ij}}{a} - \sum_{k=1}^{3} x_{kj} + (1 - q) \sum_{k=1}^{3} x_{kj},
\]

subject to

\[
0 \leq x_{ij} \leq a_{ij} a, \quad \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij} \leq p.
\]

The objection function in (14) is neither quasi-concave nor quasi-convex, which precludes a simple solution.

Searching over 72 dimensions for \(x_{ij}\) is not computationally feasible, and we resort to a combination of simulated annealing [12], gradient descent [13] and linear programming [14]. We use the simulated annealing algorithm with 200 different random feasible starting values and with the evidence-cartridge-\(h_{ij}\) index policy (described below) as the starting point, and the gradient descent algorithm with 2000 different random feasible starting values and with the evidence-cartridge-\(h_{ij}\) index policy as the starting point, and refer to the best of these solutions as \(x_{ij}^*\).

Because \(x_{ij}^*\) is not guaranteed to be the global optimum, we use linear programming to further improve the results. Let us define the function

\[
g_j(t) = \frac{t}{qa \sum_{i=1}^{3} a_{ij} + (1 - q) t},
\]

and suppose that the value of \(\sum_{i=1}^{3} x_{ij}\) is fixed at the level of \(t_j^* = \sum_{i=1}^{3} x_{ij}^*\) for each \(j\). Then
the optimization problem can be expressed as

$$\max_{x_{ij}} \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{g_j(t_j^*)h_{ij}}{a} x_{ij},$$  \hspace{1cm} (21)$$

subject to

$$\sum_{i=4}^{I} \sum_{j=1}^{J} x_{ij} = p - \sum_{j=1}^{J} t_j^*, \quad \hspace{1cm} (22)$$

$$3 \sum_{i=1}^{3} x_{ij} = t_j^*, \quad \hspace{1cm} (23)$$

$$0 \leq x_{ij} \leq a_{ij}a. \quad \hspace{1cm} (24)$$

The linear program (21)-(24) has a simple structure that allows for a closed-form solution. For each \( j \), we rank the three values of \( h_{1j}, h_{2j} \text{ and } h_{3j} \), and then allocate the capacity to the higher values of \( h_{ij} \). If we define the new subscripts \((i)\) such that \(h_{(1)j} \geq h_{(2)j} \geq h_{(3)j} \), then the optimal solution \( x_{ij}^* \) for \( i = 1, 2, 3 \) is

$$x_{(i)j}^* = \min\{a_{(i)j}a, t_j^* - \sum_{n=1}^{i-1} a_{(n)j}a\} I_{\{t_j^* \geq \sum_{n=1}^{i-1} a_{(n)j}a\}},$$  \hspace{1cm} (25)$$

where \( I_{\{x\}} \) is the indicator function of the event \( x \).

Similarly, for \( i = 4, 5, 6 \), we rank the 36 values of \( g_j(t_j^*)h_{ij} \) from highest to lowest, and then allocate the capacity to the higher values of \( g_j(t_j^*)h_{ij} \). If we define the subscripts \( i^{(1)}, \ldots, i^{(36)} \) and \( j^{(1)}, \ldots, j^{(36)} \) such that \( g_j(t_j^{(1)})h_{i^{(1)}j^{(1)}} \geq \cdots \geq g_j(t_j^{(36)})h_{i^{(36)}j^{(36)}} \), then the optimal solution \( x_{ij}^* \) for \( i = 4, 5, 6 \) is

$$x_{i^{(m)}j^{(m)}}^* = \min\{a_{i^{(m)}j^{(m)}}a, p - \sum_{j=1}^{J} t_j^* - \sum_{n=1}^{m-1} a_{i^{(n)}j^{(n)}}a\} I_{\{p - \sum_{j=1}^{J} t_j^* \geq \sum_{n=1}^{m-1} a_{i^{(n)}j^{(n)}}a\}}.$$  \hspace{1cm} (26)$$

By construction, the solution \( x_{ij}^* \) is at least as good as \( x_{ij}^* \). Although this approach does not guarantee that the global optimum is obtained, we nonetheless refer to the resulting \( x_{ij}^* \) from this method as the optimal solution. Substituting \( x_{ij}^* \) into (21) yields the optimal hit proportion

$$h(p) = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{g_j(t_j^*)h_{ij}}{a} x_{ij}^*.$$  \hspace{1cm} (27)$$

Index Policies: In an attempt to find a policy that is effective but simpler than the optimal policy, we consider five index policies of increasing complexity, where \((i, j)\) pairs
are prioritized according to an index, or score. The optimization problem (17)-(19) and its solution (25)-(26) suggest that there are three characteristics that might contribute to an effective policy: (i) giving priority to \((i, j)\) pairs with high hit probabilities \(h_{ij}\), (ii) prioritizing crime scene evidence \((i = 1, 2, 3)\) over test-fires \((i = 4, 5, 6)\), and (iii) grouping cartridges of the same type together, so that \((i, j)\) pairs are prioritized contiguously for a given value of \(j\).

Before defining the five policies, we derive the hit proportion for a generic index policy with values \(c_{ij}\). We rank the \((i, j)\) pairs by \(c_{ij}\) and define the corresponding subscripts via

\[
c_i(n) \geq \cdots \geq c_i(N),
\]

(28)

For each fixed value of \(p \leq a\), there exists a positive integer \(K \leq IJ\) such that

\[
\sum_{n=1}^{K-1} a_{i(n)}j(n) \leq p \leq \sum_{n=1}^{K} a_{i(n)}j(n).
\]

Then the values of \(x_j\) for the index policy are

\[
x_i(n)j(n) = \begin{cases} 
a_{i(n)}j(n) & n = 1, \ldots, K - 1; \\
p - \sum_{n=1}^{K} a_{i(n)}j(n) & n = K; \\
0 & n = K + 1, \ldots, IJ,
\end{cases}
\]

(29)

and the hit proportion achieved by this generic index policy is found by substituting (29) into (14). We now describe the five index policies.

The \(h_{ij}\) index policy sets \(c_{ij} = h_{ij}\), which is the probability that an arrival of this type obtains a hit.

The evidence-\(h_{ij}\) index policy prioritizes evidence over test-fires, and then ranks by \(h_{ij}\) within evidence and within test-fires. This is achieved by setting \(c_{ij} = h_{ij} + 1\) for \(i = 1, 2, 3\), and \(c_{ij} = h_{ij}\) for \(i = 4, 5, 6\).

The cartridge-\(h_{ij}\) index policy first ranks the \(J\) cartridge types by

\[
\frac{\sum_{i=1}^{I} a_{ij}h_{ij}}{\sum_{i=1}^{I} a_{ij}}
\]

which is the aggregate hit probability of type \(j\) if all cartridges of type \(j\) are processed. Within each cartridge type \(j\), we prioritize by \(h_{ij}\). This is achieved by setting \(c_{ij} = h_{ij} + M_j\), where cartridge type \(j\) has the \(M_j^{th}\) smallest value of

\[
\frac{\sum_{i=1}^{I} a_{ij}h_{ij}}{\sum_{i=1}^{I} a_{ij}}
\]

for \(j = 1, \ldots, J\).
The cartridge-evidence-$h_{ij}$ index policy is a three-level policy: it first prioritizes the $J$ cartridge types by $\sum_{i=1}^{J} a_{ij} h_{ij}$, then prioritizes evidence $(i = 1, 2, 3)$ over test-fires $(i = 4, 5, 6)$, and finally prioritizes evidence or test-fires with the same cartridge type by $h_{ij}$. The indices are defined by $c_{ij} = h_{ij} + 2M_j + 1$ for $i = 1, 2, 3$, and $c_{ij} = h_{ij} + 2M_j$ for $i = 4, 5, 6$.

The evidence-cartridge-$h_{ij}$ index policy first prioritizes evidence $(i = 1, 2, 3)$ over test-fires $(i = 4, 5, 6)$, and then within evidence ranks cartridge types by $\sum_{i=1}^{3} a_{ij} h_{ij}$. When processing evidence of the same cartridge type or processing test-fires, we prioritize by $h_{ij}$.

Evidence Random Policy: In this policy, all evidence (crime types $i = 1, 2, 3$) is prioritized over all test-fires $(i = 4, 5, 6)$. However, in contrast to the index policies, we do not use any other information to rank within evidence or within test-fires, and simply allocate the capacity randomly within evidence and within test-fires. When $p \leq \sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} a$, only evidence is processed and $x_{ij} = \frac{a_{ij}}{\sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij}} p$ for $i = 1, 2, 3$ and $x_{ij} = 0$ for $i = 4, 5, 6$. The hit proportion in this case is given by

$$h(p) = \frac{\left( \frac{p}{\sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} a} \right)^2}{q + (1 - q) \frac{p}{\sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij}}} \sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} h_{ij}. \quad (30)$$

When $\sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} a < p \leq a$, all evidence and some test-fires are processed and $x_{ij} = a_{ij} a$ for $i = 1, 2, 3$ and $x_{ij} = \frac{a_{ij}(p - \sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} a)}{\sum_{i=4}^{4} \sum_{j=1}^{J} a_{ij}}$ for $i = 4, 5, 6$. The hit proportion in this case is given by

$$h(p) = \sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} h_{ij} + \frac{\sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} h_{ij} p - \sum_{i=1}^{3} \sum_{j=1}^{J} a_{ij} a}{a}. \quad (31)$$

Stockton and other municipalities [4] often give priority to cartridges related to homicides. The final two policies are constrained versions of the random policy and optimal policy introduced above, where it is required that arrivals of homicide evidence $(i = 1$ in
Table [1] are given higher priority than homicide test-fires \((i = 4)\), which in turn are given higher priority than the other four crime types. The two constrained policies vary by how they allocate within crime type 1, within crime type 4, and within crime types 2, 3, 5 and 6, and they are referred to as the homicide-random policy and the homicide-optimal policy, respectively.

**Homicide-Random Policy:** In this policy, allocation decisions within crime type 1, within crime type 4, and within crime types 2, 3, 5 and 6 are made in a random manner; i.e., within each of the three groupings, each \((i, j)\) pair is processed in proportion to \(a_{ij}\). The analysis of this policy breaks into three cases, depending on the amount of available capacity. When \(0 \leq p \leq a \sum_{j=1}^{J} a_{1j}\), then all available capacity is allocated to homicide evidence \((i = 1)\). This capacity is allocated randomly to each cartridge type, and hence each cartridge type is processed in proportion to its fraction of all homicide evidence, i.e.,

\[
x_{1j} = \frac{a_{ij}}{\sum_{j=1}^{J} a_{1j}} p \quad \text{and} \quad x_{ij} = 0 \quad \text{for} \quad i \neq 1, 4.
\]

It follows that the hit proportion in this case is

\[
h(p) = \sum_{j=1}^{J} \frac{a_{1j} p h_{1j}}{a} \frac{\sum_{k=1}^{3} a_{1k} p}{q a \sum_{j=1}^{J} a_{ij} + (1 - q) \sum_{k=1}^{J} a_{1k} p},
\]

\[
= \sum_{j=1}^{J} a_{1j}^2 h_{1j} \frac{\left( \frac{p}{a \sum_{k=1}^{J} a_{1k}} \right)^2}{q \sum_{j=1}^{J} a_{ij} + (1 - q) \sum_{k=1}^{J} a_{1k} p}.
\]

When \(a \sum_{j=1}^{J} a_{1j} < p \leq a \sum_{j=1}^{J} (a_{1j} + a_{4j})\), then all homicide evidence and some homicide test-fires are processed. In this case, \(x_{1j} = a_{1j} a, \ x_{4j} = \frac{a_{4j}}{\sum_{k=1}^{3} a_{4k}} (p - \sum_{k=1}^{J} a_{1k} a), \) and \(x_{ij} = 0\) for \(i \neq 1, 4\). The hit proportion in this case is

\[
h(p) = \frac{1}{a} \sum_{j=1}^{J} \left( (a_{1j} a h_{1j} + \frac{a_{4j}}{\sum_{k=1}^{3} a_{4k}} (p - \sum_{k=1}^{J} a_{1k} a) h_{4j}) \frac{a_{1j}}{q \sum_{j=1}^{J} a_{ij} + (1 - q) a_{1j}} \right).
\]

Finally, when \(a \sum_{j=1}^{J} (a_{1j} + a_{4j}) \leq p \leq a\), then all homicides and some non-homicides are processed, and \(x_{1j} = a_{1j} a, \ x_{4j} = a_{4j} a, \) and \(x_{ij} = \frac{a_{ij}}{\sum_{l \neq 1, 4} a_{ij}} (p - \sum_{k=1}^{J} (a_{1k} + a_{4k}) a)\) for \(i \neq 1, 4\).
In this case, the hit proportion is

\[
h(p) = \sum_{j=1}^{J} \left( a_{1j} h_{1j} + a_{4j} h_{4j} + \frac{\sum_{i \neq 1,4} a_{ij} h_{ij}}{\sum_{i \neq 1,4} a_{ij}} \frac{p - \sum_{j=1}^{J} (a_{1j} + a_{4j}) a}{a} \right) - \frac{a_{1j} a + (a_{1j} + a_{3j}) (p - \sum_{j=1}^{J} (a_{1j} + a_{4j}) a)}{\sum_{i \neq 1,4} a_{ij}} \right) \left( qa \sum_{k=1}^{3} a_{kj} + (1 - q) \left( a_{1j} a + \frac{(a_{1j} + a_{3j}) (p - \sum_{j=1}^{J} (a_{1j} + a_{4j}) a)}{\sum_{i \neq 1,4} a_{ij}} \right) \right). \tag{36}
\]

**Homicide-Optimal Policy:** In this policy, we optimally allocate capacity within crime type 1, within crime type 4, and within crime types 2, 3, 5, and 6. We have the same three cases as in the homicide-random policy, but now we solve an optimization problem in each case. When \(0 \leq p \leq a \sum_{j=1}^{J} a_{1j}\), then \(x_{ij} = 0\) \(\forall i \neq 1\), and by equation (14) the optimization problem is

\[
\max_{x_{1j}} \sum_{j=1}^{J} h_{1j} \frac{x_{1j}^2}{a} - \frac{qa \sum_{l=1}^{3} a_{ij} + (1 - q) x_{1j}}{a_{1j} a}, \tag{37}
\]

subject to

\[
\sum_{j=1}^{J} x_{1j} = p, \tag{38}
\]

\[
0 \leq x_{1j} \leq a_{1j} a. \tag{39}
\]

Because problem (37)-(39) is the maximization of a convex function over a convex polyhedron, the optimum is achieved at one of the vertices of the polyhedron (38)-(39) [13], where for some \(k = 1, \ldots, J\), \(x_{1j} = 0\) or \(a_{1j} a\) for \(j \neq k\), and \(x_{1k} = p - \sum_{j \neq k} x_{1j}\). We find the optimal solution by comparing the value of the objective function (37) for all of these vertices.

When \(a \sum_{j=1}^{J} a_{1j} < p \leq a \sum_{j=1}^{J} (a_{1j} + a_{4j})\), we have \(x_{1j} = a_{1j} a\) and \(x_{ij} = 0\) \(\forall i \neq 1, 4\). Substituting these values into (14) and discarding terms that do not depend on \(x_{4j}\) yields the linear program

\[
\max_{x_{4j}} \sum_{j=1}^{J} h_{4j} \frac{x_{4j} a_{1j}}{a} - \frac{qa \sum_{l=1}^{3} a_{ij} + (1 - q) a_{4j} a}{a_{4j} a}, \tag{40}
\]

subject to

\[
\sum_{j=1}^{J} x_{4j} = p - \sum_{j=1}^{J} a_{1j} a, \tag{41}
\]

\[
0 \leq x_{4j} \leq a_{4j} a, \tag{42}
\]

20
which is a constrained knapsack problem that is easy to solve. Finally, when \( p \geq a \sum_{j=1}^{J} (a_{1j} + a_{4j}) \), then \( x_{1j} = a_{1j}a \) and \( x_{4j} = a_{4j}a \). Substituting into (14) yields the optimization problem

\[
\max_{x_{ij}, i \neq 1, 4} \sum_{j=1}^{J} \left( a_{1j}h_{1j} + a_{4j}h_{4j} + \sum_{i \neq 1, 4} \frac{h_{ij}x_{ij}}{a} \right) (a_{1j}a + x_{2j} + x_{3j})
\]

subject to

\[
\sum_{i \neq 1, 4} \sum_{j=1}^{J} x_{ij} = p - \sum_{j=1}^{J} (a_{1j} + a_{4j})a,
\]

\[
0 \leq x_{ij} \leq a_{ij}a \quad \text{for} \quad i \neq 1, 4
\]

which can be solved in the same way that we derived the optimal policy.

**Analysis of Useful Hits**

We now take the processing capacity \( p \) and the capacity allocation policy as given – i.e., we know exactly which arrivals will be processed, and the resulting hit proportion – and focus on the precise order in which these arrivals are processed. Recall that a useful hit is one that occurs before its corresponding criminal case is closed. Referring to notation introduced earlier, a hit is useful if \( W < C \), where \( W \) is the delay from the event date to the process completion date, and \( C \) is the delay from the event date to the case closure date.

Queueing theory does not allow for a reliable mathematical expression for the entire distribution of \( W \), which is needed to estimate the proportion of hits that are useful. Hence, in both the in-house scenario and external lab scenario, we estimate the probability density function (PDF) of \( W \) by simulating a queueing system with the parameters in Table 7, where the interarrival times and service times are gamma random variables. The PDF of \( W \) also depends on the priority policy that is used. For the in-house scenario, we consider four policies (Table 9): FCFS, LCFS, homicide-FCFS and homicide-LCFS, where homicide means that homicides (crime types \( i = 1 \) and \( 4 \) in Table 1) are processed before non-homicides (i.e., non-homicides are only processed when there are no homicide cartridges in the queue), and either FCFS or LCFS is used to prioritize among the homicides in queue and the non-homicides in queue. For the external lab scenario, we only consider LCFS and FCFS because most of what the external lab receives represents each metropolis’s highest-priority evidence.
To estimate the PDF of $W$ under the in-house scenario and any priority policy, we start the simulation with the steady-state number of customers in queue (which is $aE[W] = 74$ by Little’s formula [9]), each of whom has spent $\frac{av}{p(p-a)} = 17.8$ days in queue based on equation (5). We run the simulation for 500 years, which generates $\approx 75,000$ waiting times, and discard the first 10,000 waiting times. This process is repeated five times and the distribution of $W$ is taken to be the average of the five distributions, which are all similar to each other. We perform the same procedure for the external lab scenario, except that we run each simulation for 1500 years, discarding the first $10^5$ waiting times out of the $\approx 5 \times 10^6$ generated.

Let $f(w)$ and $g(c)$ be the PDFs of $W$ and $C|C < \infty$, respectively, and let $F(w)$ be the CDF of $W$. Then the proportion of hits that are useful is

$$P(W < C) = 0.387 + 0.613 \int_0^\infty F(c)g(c) \, dc.$$  \hspace{1cm} (46)

To estimate the integral in (46), we take $10^6$ random samples from $f(w)$ and $10^6$ random samples from $g(c)$, and compute the proportion for which $w < c$. We repeat this procedure 100 times, so that the 95% half confidence interval for the mean of the 100 proportions is $\approx 10^{-3}$.

**Results**

**Hit proportions**

The potential hit proportion from equation (8) is 0.127. By construction, all ten capacity allocation policies in Table 8 achieve a hit proportion of 0 when $p = 0$ and a hit proportion of 0.127 when $p = 4.10$/day, which is the value of the arrival rate $a$ (Fig. 8). As an aggregate measure of performance for the capacity allocation policies, we compute the normalized area under the curve (AUC), which is the AUC of the hit proportion function $h(p)$ in Fig. 8 divided by the total area in Fig. 8, which is 0.5207 (the potential hit proportion 0.127 times the arrival rate 4.10). The random policy, while performing better than the hit
proportion, \((\frac{p}{4.10})^2 \cdot 0.127\), that would be achieved if no arriving cartridges had more than one match in the database (i.e., \(q = 1\) in (20)), has a hit proportion function \(h(p)\) that is convex and falls well below the linear function \((\frac{p}{4.10}) \cdot 0.127\) (Fig. 8), which is consistent with its normalized AUC being less than 0.5 (Table 8).

In contrast, the optimal policy performs better than linearly, i.e., the hit proportion is greater than \((\frac{p}{4.10}) \cdot 0.127\) and the normalized AUC is greater than 0.5, and more than doubles the hit proportion of the random policy when approximately half of the arrivals are processed (Fig. 8). The optimal policy is complicated and – in contrast to the five index policies – the allocations \(x_{ij}\) are not nondecreasing in the processing capacity \(p\).

Although the five index policies have varying levels of complexity (Table 8), they all have quite similar performance and are somewhat close to optimal, as seen in Fig. 8 and in the normalized AUC values in Table 8. Moreover, none of the five index policy curves dominate any of the others (Fig. 8), with their relative ranking depending on the value of \(p\).

Recall that there are 72 \((i, j)\) pairs, half of which are evidence and half test-fires. In the \(h_{ij}\) index policy, 24 of the largest 36 \(h_{ij}\) values are evidence \((i = 1, 2, 3)\), which partially explains why the \(h_{ij}\) index policy and the evidence-\(h_{ij}\) index policy achieve very similar performance.

The cartridge-\(h_{ij}\) index policy performs very similarly to the first two index policies, but takes an entirely different approach. Here the hit proportion function \(h(p)\) consists of 12 different segments – one for each cartridge type – each having the shape of the curve generated by the random policy. Because the cartridge types are ranked by their aggregate potential hit proportion, the slope of each segment gets successively smaller as we progress along the curve, and a piecewise-linear curve (not shown in Fig. 8) that connects the 13 endpoints of the 12 curve segments would be concave; more generally, relative to the convex performance of the random policy, the optimal policy and all five index policies nearly achieve concave performance.
The two three-level index policies, by incorporating both approaches of prioritizing evidence and grouping cartridge types of the same caliber, outperform the other three index policies, with the evidence-cartridge-\(h_{ij}\) index policy, which prioritizes evidence before grouping cartridge types, achieving the highest normalized AUC and cutting the optimality gap (as measured by the normalized AUC) by at least 60% relative to the first three index policies.

Moreover, observing that the evidence-cartridge-\(h_{ij}\) index policy outperforms the evidence-\(h_{ij}\) index policy whereas the cartridge-\(h_{ij}\) and \(h_{ij}\) index policies perform very similarly, we conclude that grouping cartridge types helps only when evidence is given priority over test-fires. Similarly, because the cartridge-evidence-\(h_{ij}\) index policy outperforms the cartridge-\(h_{ij}\) index policy but the evidence-\(h_{ij}\) index policy does not outperform the \(h_{ij}\) index policy, we conclude that prioritizing evidence over test-fires is more helpful when cartridge types are grouped. That is, these two approaches – prioritizing evidence over test-fires and grouping cartridge types – appear to be synergistic.

Turning to the final three policies, we see that the evidence-random policy consists of two convex segments that connect at \(p = 2.44/\text{day}\), which is the total arrival rate of crime scene evidence. The slope of the left segment is steeper than the slope of the right segment because evidence has a higher hit proportion than test-fires; evidence makes up \(\approx 60\%\) of arrivals and achieves \(\approx 80\%\) of the hits. Hence, the evidence-random policy is not far from optimal when \(p = 2.44/\text{day}\), but otherwise can incur significant suboptimality.

Because homicide-related crime types \((i = 1, 4)\) have a combined hit proportion that is similar to the overall average hit proportion (Fig. 1a), the homicide-random policy achieves performance that is nearly identical to the performance of the random policy. For \(p < 0.60/\text{day}\), which is the homicide-related arrival rate, the homicide-optimal policy processes only homicide-related evidence and performs similarly to the two random policies. For \(p\) between 0.60/\text{day} and 1.8/\text{day}, the performance curve of the homicide-optimal policy is
roughly parallel to the curve of the optimal policy. The homicide-optimal policy performs very well for \( p > 1.8/\text{day} \).

**Useful hits**

For the in-house scenario, we compute the proportion of useful hits for the four priority policies that are described in Table 9. A visual inspection of the CDFs of the waiting time \( W \) for each of these priority policies and for the conditional case closing time \( C|C < \infty \) (Fig. 9a) suggest that the proportion of useful hits will be highest for LCFS, lowest for FCFS, and very similar for the homicide-constrained and unconstrained versions of these policies; these observations follow because when the case closing times are finite, they are often very small, and LCFS has a higher probability than FCFS of achieving a very small waiting time. Although this is confirmed in Table 10, the increase in the proportion of hits that are useful, from 0.568 for FCFS to 0.633 for LCFS, is modest.

For the external lab scenario, the waiting time CDFs of FCFS and LCFS (along with the CDF of \( C|C < \infty \)) appear in Fig. 9b, and the benefit due to switching from LCFS to FCFS is again modest, from 0.463 to 0.514 (Table 10). Because waiting times are typically longer at the external lab than in-house, the proportion of useful hits is smaller in the external lab scenario.

**Discussion**

**Results**

We have three main results. The first result can be seen most easily from equation (16): when \( q = 1 \), which means that no newly acquired cartridges have more than one match in the database, and arrivals are processed without regard to crime type or cartridge type, e.g. using FCFS, then – when the capacity is less than the arrival rate – the number of hits is proportional to \( \left( \frac{\text{capacity}}{\text{arrival rate}} \right)^2 \). In virtually all manufacturing and service operations, the potential output is proportional to \( \frac{\text{capacity}}{\text{arrival rate}} \) (when the capacity is less than the arrival rate); e.g., if you double the number of cashiers at a supermarket or double the number of
machines in a manufacturing facility, then you double the potential output. In contrast, if \( q = 1 \), if half of the arriving cartridges are processed, then we achieve only one-quarter of the potential number of hits, and if 10\% of the arrivals are processed, we achieve only 1\% of the potential number of hits. Hence, the marginal return to adding capacity is increasing (i.e., the hit proportion function \( h(p) \) is convex in \( p \)) rather than constant, as in the linear case.

The value of \( \frac{\text{capacity}}{\text{arrival rate}} \) varies widely among different municipalities in the U.S. [4]. Boston [3] and Stockton process all of their evidence (i.e., \( p \geq a \)), and achieve good performance. At the other extreme, some cities enter very little evidence and – as predicted by equation (16) – generate very few hits. To the extent that poor-performing cities linearly extrapolate what their performance would be if they increased processing capacity, they can be grossly underestimating the efficacy of ballistic imaging systems. We note that the quadratic performance underlying equation (16) also holds for DNA matching operations, which has also been plagued – although to a lesser extent – by lack of funding, leading to undercapacitated systems and suboptimal performance [15].

The best hope for increasing processing capacity in many cities is to bring at least some of their processing in house, as Stockton did. See [16] for a discussion of other benefits from keeping crime labs within law enforcement agencies.

The second main result is that the number of hits can be significantly increased – with performance changing from approximately quadratic to superlinear and nearly concave (Fig. 8) – by using a rather simple index policy. All five index policies perform somewhat close to optimal and can approximately double the hit proportion when only half of the arrivals are processed. Some of the improvement from the index policies is due to prioritizing evidence over test-fires, and some is due to grouping the evidence into cartridge types (as opposed to ranking them), which converts one convex curve into 12 convex segments (Fig. 8). Moreover, these two approaches are synergistic. However, prioritizing the cartridge types by
their aggregate hit proportion also helps, and the top priority cartridge types in Fig. 1b, \( j = 8 \) and 10, are known to be popular in Stockton among Black and Hispanic gangs, respectively.

While some municipalities appear to give high priority to certain cartridge types (e.g., automatic or semi-automatic firearms, Table 7 in [4]), or to homicides, or to evidence over test-fires, we are not aware of any cities that follow a policy similar in form to any of these index policies. We found that only prioritizing evidence over test-fires can lead to significant suboptimality, and prioritizing homicide-related arrivals over nonhomicide arrivals can decrease the number of hits generated if the proportion of arriving cartridges that are processed is less than 0.4 (Fig. 8). Although the index policies are not optimal (Fig. 8), their simplicity and the fact that their \( x_{ij} \) values are nondecreasing in the total capacity make them easier to implement (and more robust) than the optimal policy in a setting where processing capacity is increased slowly over many years. While it is conceivable that criminals could strategically counteract an index policy that groups cartridge types by switching to unprocessed cartridge types, this seems unlikely given the system’s lack of transparency from a criminal’s viewpoint, and the fact that strategic behavior has not been observed elsewhere; e.g., criminals in Boston did not increase their use of revolvers, which do not eject cartridges, as a result of the implementation of a ballistic imaging system [5].

For a municipality that is not in a position to reliably estimate \( h_{ij} \) due to a lack of historical NIBIN use (and hence a lack of data), a less data-intensive policy would be to prioritize evidence over test-fires, group the evidence into cartridge types, and then prioritize the cartridge types by their aggregate hit proportions, which would be easier to reliable estimate than the individual \( h_{ij} \) values, or – if no data are available – by their perceived popularity.

The third main result is a negative one. Although switching from FCFS to LCFS increases the proportion of hits that are detected before the corresponding criminal case is closed, the improvements – from 0.568 to 0.633 in the in-house scenario and from 0.463 to
0.514 in the external lab scenario – are rather modest. Given that customers in queueing systems perceive LCFS as extremely unfair [17], this modest increase is probably not worth pursuing from a policy viewpoint.

**Limitations**

In the hit analysis, one limitation is that there is no way to estimate the false negative rate; i.e., we do not know how many true hits were missed by the firearms technicians. Nonetheless, although the details about crime types and cartridge types will vary from city to city, we believe that our first two main results are quite robust. However, given that 72 $h_{ij}$ values had to be estimated, we did not have sufficient data to split the data in two portions, using the early years to estimate the parameter values and using the later years to assess the performance of our policies. To the extent that $a_{ij}$ and $h_{ij}$ values may change slowly over time and that rare $(i,j)$ pairs may be inaccurately estimated, our improvements in Fig. 8 should be viewed as upper bounds on the improvements that would be achieved by these policies in practice.

In contrast, the third main result about useful hits is based on a simplified queueing model that does not capture many of the complexities of the actual system. The precise prioritization of the cartridges in queue was not directly observable and was somewhat complex (e.g., criminal cases coming to trial were given high priority when requested by prosecutors), and in estimating the model parameters via equations (5)-(7) we assume that the prioritization is independent of the hit proportion (i.e., the waiting time of a cartridge was independent of the likelihood that the cartridge would generate a hit). However, the hit proportions for criminal types $i = 1, 2, 3$ are slightly higher than the hit proportions for crime types $i = 4, 5, 6$ in Fig. 1a, and Stockton often gave non-homicide test-fires $(i = 5, 6)$ lower priority.

The resulting values for the coefficients of variation $c_a$ and $c_s$ in Table 7 are much larger than the values of $c_a = c_s = 1$ that are the benchmark in the queueing literature and
that correspond to exponential interarrival and processing times. Although $c_a = c_s = 1$ is perceived as possessing substantial variability, there are telecommunications settings where the interarrival times are estimated to have fat-tailed distributions with infinite variance \cite{18}.

In addition, our huge values of $E[W]$ and $c_a$ and $c_s$ lead to difficulties in reliably estimating the probability distribution of the waiting time, even after simulating the queueing system for hundreds of years.

There are several possible reasons for the large values of $c_a$ and $c_s$. The incidence of gun crimes can be very bursty, with several in a day, followed by several weeks without one, and some gun crimes generate multiple cartridges. In addition, several upstream phenomena (the evidence storage room is locked over the weekend, DNA processing of evidence is often performed prior to ballistic imaging and is delivered in bulk) inadvertently increase the variability in the interarrival times. Processing times are somewhat bimodal (and hence highly variable) because the time to search for matches to arriving cartridges that do not generate any hits is much shorter (e.g., 30 minutes) than the time to search for matches to arriving cartridges that do generate hits (e.g., four hours).

Moreover, the calculation of useful hits depends not just on the probability distribution of waiting times in the queue, but also on the probability distribution of the case closing time \cite{46}. The only data on case closing times that we could locate are for homicides, which comprise only 13.9% of all arriving cartridges. However, the comparison of FCFS vs. LCFS is extreme (i.e., generating much different waiting time distributions), and the broad insights from this comparison do not rely on a well-validated queueing model. That is, our qualitative conclusion – that LCFS achieves a higher proportion of hits that are useful than FCFS – is likely robust, although the quantitative results in Table 10 should not be considered reliable. In summary, because we obtained a negative result (i.e., LCFS achieves only a modest improvement over FCFS) using our best estimates of the queueing system parameters, we did not deem it important to perform sensitivity analysis for this portion of
the study. That is, we would recommend LCFS over the status quo of FCFS only if LCFS increased the proportion of hits that are useful in a substantial and robust manner, and our results in Table 10 preclude this possibility.

Also, we are implicitly assuming that a hit is useful if the unconfirmed hit occurs before the criminal case is closed. In Stockton, unconfirmed hits are often passed along (as unconfirmed) to investigators and prosecutors.

**Conclusion**

Analyzing data from a city (Stockton, CA) that processes all of its ballistic images provides us with an uncensored view that allows us to counterfactually estimate what would happen if processing capacity was limited. We obtain three main results: when capacity is limited – as it is in most U.S. municipalities – the number of hits increases according to $(\frac{\text{capacity}}{\text{arrival rate}})^2$, which is much smaller than in traditional service and manufacturing operations, where output increases according to $\frac{\text{capacity}}{\text{arrival rate}}$. Second, simple index policies that give priority to evidence over test-fires, and group cartridges by their cartridge type and allocate capacity to only the higher-ranking cartridge types, significantly increases the number of hits (e.g., when the processing capacity is half of the arrival rate, the number of hits can be doubled, Fig. 8). Finally, LCFS increases the proportion of hits that are useful by only 0.05 relative to FCFS, which is probably insufficient to offset the severe increase in perceived inequity generated by LCFS. In summary, our analysis reinforces and refines the point in [19] that processes and people are needed – not just technology – to improve the performance of ballistic imaging.

**Acknowledgment**

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References


Quebec, Canada: Forensic Technology WAI, Inc., 2010.
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Table 1: The crime types $i = 1, \ldots, 6$ and the cartridge types $j = 1, \ldots, 12$. 
Table 2: The transpose of the matrix $a_{ij}$, which is the proportion of arriving cartridges that are of crime type $i = 1, \ldots, 6$ and cartridge type $j = 1, \ldots, 12$. 

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Table 2: The transpose of the matrix $a_{ij}$, which is the proportion of arriving cartridges that are of crime type $i = 1, \ldots, 6$ and cartridge type $j = 1, \ldots, 12$. 


Table 3: The transpose of the matrix $h_{ij}$, which is the proportion of arriving cartridges of crime type $i = 1, \ldots, 6$ and cartridge type $j = 1, \ldots, 12$ that generate at least one hit.

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Table 4: The transpose of the matrix of the number of NIBIN entries of crime type $i = 1, \ldots, 6$ and cartridge type $j = 1, \ldots, 12$ during July 26, 2011 to February 20, 2013 and from June 18, 2013 to February 27, 2015. The matrix entries with less than 30 arrivals are in boldface.
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<td>0</td>
<td>19</td>
<td>21</td>
<td>0</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: The number $n_{ijk}$ of arriving cartridges of type $(i, j)$ that match with type $(k, j)$ in the database.
Table 6: The transpose of the matrix of the values in equation (3), which are the conditional probabilities that when an arriving cartridge of type \((i, j)\) has a hit, its match is of type \((k, j)\) for \(k = 1, 2, 3\) (independent of \(i\)).

<table>
<thead>
<tr>
<th>(j)</th>
<th>(k = 1)</th>
<th>(k = 2)</th>
<th>(k = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>2</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>3</td>
<td>0.0946</td>
<td>0.3072</td>
<td>0.5982</td>
</tr>
<tr>
<td>4</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>5</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>6</td>
<td>0.1248</td>
<td>0.5443</td>
<td>0.3309</td>
</tr>
<tr>
<td>7</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>8</td>
<td>0.2106</td>
<td>0.3931</td>
<td>0.3963</td>
</tr>
<tr>
<td>9</td>
<td>0.1348</td>
<td>0.4534</td>
<td>0.4118</td>
</tr>
<tr>
<td>10</td>
<td>0.1029</td>
<td>0.3782</td>
<td>0.5189</td>
</tr>
<tr>
<td>11</td>
<td>0.2231</td>
<td>0.3513</td>
<td>0.4256</td>
</tr>
<tr>
<td>12</td>
<td>0.0895</td>
<td>0.3749</td>
<td>0.5356</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
<td>In-House</td>
<td>External Lab</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>---------------</td>
<td>--------------</td>
</tr>
<tr>
<td>(a)</td>
<td>arrival rate of evidence</td>
<td>4.10/day</td>
<td>9.39/day</td>
</tr>
<tr>
<td>(p)</td>
<td>processing rate of evidence</td>
<td>5.55/day</td>
<td>5.55/day</td>
</tr>
<tr>
<td>(m)</td>
<td>number of servers</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(c_a^2)</td>
<td>squared coefficient of variation of interarrival times</td>
<td>35.05</td>
<td>735</td>
</tr>
<tr>
<td>(c_s^2)</td>
<td>squared coefficient of variation of processing times</td>
<td>35.05</td>
<td>735</td>
</tr>
<tr>
<td>(P(C &lt; \infty))</td>
<td>proportion of cases that are closed</td>
<td>0.613</td>
<td>0.613</td>
</tr>
<tr>
<td>(C</td>
<td>C &lt; \infty)</td>
<td>delay from crime to case closure for closed cases</td>
<td>LN(0.176,2.62)</td>
</tr>
</tbody>
</table>

Table 7: The queueing model parameters and their values under the in-house scenario and external lab scenario. \(LN(\mu, \sigma)\) denotes the lognormal distribution with parameters \(\mu\) and \(\sigma\).
Table 8: The ten capacity allocation policies along with their aggregate performance, as measured by the normalized Area Under the Curve (AUC) of the hit proportion vs. processing capacity curve in Fig. 8. The AUCs are normalized by dividing by the total area in Fig. 8, which is 0.5207 (the potential hit proportion 0.127 times the arrival rate 4.10).
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCFS</td>
<td>first-come first-served</td>
</tr>
<tr>
<td>LCFS</td>
<td>last-come first-served</td>
</tr>
<tr>
<td>Homicide-FCFS</td>
<td>homicides processed before non-homicides; homicides use FCFS; non-homicides use FCFS</td>
</tr>
<tr>
<td>Homicide-LCFS</td>
<td>homicides processed before non-homicides; homicides use LCFS; non-homicides use LCFS</td>
</tr>
</tbody>
</table>

Table 9: The priority queueing policies. The two homicide policies process cartridges in queue of crime types $i = 1$ and $i = 4$ before evidence of crime types $i = 2, 3, 5$ and $6$. 
### Table 10: The performance of the priority queueing policies in the in-house and external lab scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Priority Queueing Policy</th>
<th>Proportion of Hits that are Useful</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-house</td>
<td>FCFS</td>
<td>0.568</td>
</tr>
<tr>
<td>In-house</td>
<td>LCFS</td>
<td>0.633</td>
</tr>
<tr>
<td>In-house</td>
<td>Homicide-FCFS</td>
<td>0.582</td>
</tr>
<tr>
<td>In-house</td>
<td>Homicide-LCFS</td>
<td>0.628</td>
</tr>
<tr>
<td>External lab</td>
<td>FCFS</td>
<td>0.463</td>
</tr>
<tr>
<td>External lab</td>
<td>LCFS</td>
<td>0.514</td>
</tr>
</tbody>
</table>
Figure Legends

Fig. 1. The average hit proportion for (a) crime type $i = 1, \ldots, 6$, and (b) cartridge type $j = 1, \ldots, 12$. The overall average hit proportion is 0.127 (- -).

Fig. 2. The empirical cumulative distribution function and the geometric cumulative distribution function derived by maximum likelihood estimation, for the number of hits for arriving cartridges that generate at least one hit.

Fig. 3. The number of arriving cartridges (according to the event date) in each year.

Fig. 4. The mean daily number of cartridges that were entered into NIBIN over the 300-day period that begins on the day on the horizontal axis.

Fig. 5. The histogram of the time interval between the event date and the NIBIN entry date for all cartridges that had an event date between December 25, 2012 and February 24, 2015, where (b) is the same as (a), but is truncated at 100 days.

Fig. 6. The histogram of the time interval between the NIBIN entry date and the completion date for all cartridges that had an event date between December 25, 2012 and February 24, 2015.

Fig. 7. The empirical CDF [11] and three fitted CDFs for the case closing time $C$.

Fig. 8. The hit proportion results for the ten capacity allocation policies in Table 8.

Fig. 9. (a) For the in-house scenario, the CDFs of the waiting times under FCFS, LCFS, homicide-FCFS and homicide-LCFS, and the CDF of the closing times for closed cases ($C|C < \infty$). (b) For the external lab scenario, the CDFs of the waiting times under FCFS and LCFS, and the CDF of the closing time for closed cases.
Fig. 1

(a)

(b)
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 8
Fig. 9